9/26/16 "The will to exceed is important, but whats more important is the will to prepare"
-Bobby Knight

HW: "The Derivative and Tangent Line Problem HW" #1-5
Test 2 on Wednesday 10/19

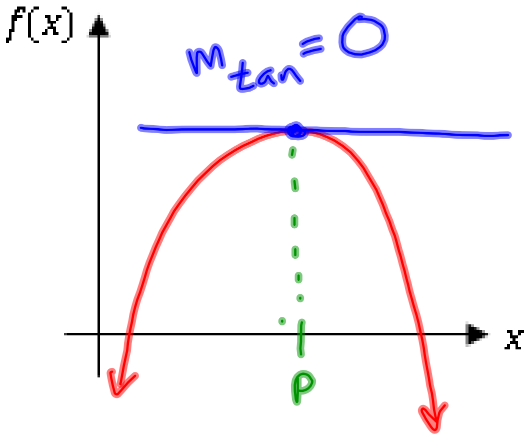
AIM: How do we find Derivatives?
Warm Up:

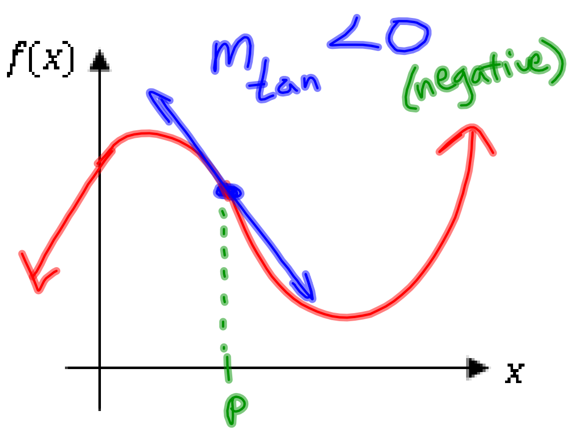
The Derivative and the Tangent Line Problem

Analytically: The definition of a derivative of f at x is the difference quotient (slope generator):

$$f'(x) = m_{tan} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Graphically: The derivative of a function at a point p is the slope of a tangent line to the graph of f at p.





Numerically: The derivative at a point is the *limit* of slopes of the secant lines or the *limit* of the difference quotient.

The derivative at a point $x = a$ is found by:

$$f'(a) = m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or by $$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.
For all a for which the limit exists,
$f'(x)$ is a function of a.

Pull

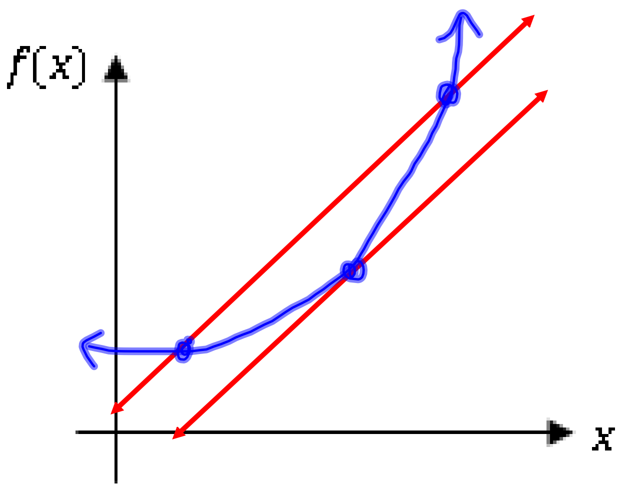
A function is differentiable at a given value of x if you can take the derivative of the function at that x value. In other words, f(x) is differentiable at x=c if f '(c) exists. A function that is differentiable at every point of its domain has a derivative.

NOTATION FOR DERIVATIVES:

$f'(x)$	$\dfrac{dy}{dx}$	y'
$D_x[y]$	$\dfrac{d}{dx}[f(x)]$	$\dfrac{df}{dx}$

Using Limit Notation:

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$



EX #1: Use the definition of the derivative to find the slope of the graph $f(x)=3x-2$. Find $f'(-3)$. Is the derivative a function ?

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{3(x+h)-2-(3x-2)}{h}=\frac{\cancel{3x}+3h\cancel{-2}\cancel{-3x}\cancel{+2}}{h}=\frac{3\cancel{h}}{\cancel{h}}$$

$f'(x)=3$ therefore $f'(-3)=3$

horizontal line is a function $\therefore$
$f'(x)$ is a funtion

EX #2: Find the slope of the tangent lines to the graph of $f(x)=x^2-5$ at the points $(-2,-1)$ and $(1,-4)$.

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^2-5-(x^2-5)}{h}$$

$$=\lim_{h\to 0}\frac{\cancel{x^2}+2xh+h^2\cancel{-5}-\cancel{x^2}\cancel{+5}}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2}{h}=\lim_{h\to 0}2x+h=2x+0$$

Derivative $\longrightarrow f'(x)=2x$

@ $(-2,-1)$ $f'(-2)=2(-2)=-4$

@ $(1,-4)$ $f'(1)=2(1)=2$

EX #3: For the function, $f(x) = x^3 - 2x$, find the derivative and evaluate the derivative at $x = 2, 0,$ and -1.

EX #4: Find $f'(x)$ for $f(x) = -\sqrt{x}$. Then find the slopes of the graph at the point $(4, -2)$.
What is the behavior of the graph at (0, 0) ?

$$f'(x) = \lim_{h \to 0} \frac{-\sqrt{x+h} - (-\sqrt{x})}{h}$$

$$\frac{-\sqrt{x+h} + \sqrt{x}}{h} \cdot \frac{-\sqrt{x+h} - \sqrt{x}}{-\sqrt{x+h} - \sqrt{x}} = \frac{x+h-x}{h(-\sqrt{x+h}-\sqrt{x})} =$$

$$\frac{\cancel{h}}{\cancel{h}(-\sqrt{x+h}-\sqrt{x})} = \frac{1}{-\sqrt{x+h}-\sqrt{x}} = \frac{1}{-\sqrt{x}-\sqrt{x}} = \frac{1}{-2\sqrt{x}}$$

$h \to 0$

$$f'(x) = \frac{1}{-2\sqrt{x}}$$

@ $(4, -2)$ $f'(4) = \frac{1}{-2\sqrt{4}} = -\frac{1}{4}$

⊛ at $(0,0)$
The graph ends ∴ The derivative DNE
$$f'(0) = \frac{1}{-2\sqrt{0}} = \frac{1}{0} \text{ undefined}$$